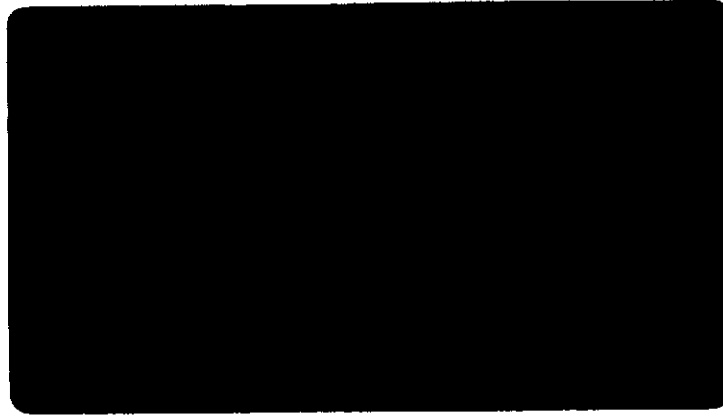


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**Mechanical Engineering Department**  
**College of Engineering**  
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**Clemson, South Carolina**



OPTIMIZATION OF STRUCTURES  
TO SATISFY A FLUTTER VELOCITY CONSTRAINT  
BY USE OF QUADRATIC EQUATION FITTING

by

Saifee K. Motiwalla

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|----------|--------------------------|----------------|
| Approved | <u>C. S. Ruffell</u>     | <u>1-18-73</u> |
|          | In Charge of Major       | Date           |
| Approved | <u>[Signature]</u>       | <u>2/6/73</u>  |
|          | Head of Major Department | Date           |
| Approved | <u>M. W. Davis</u>       | <u>1-18-73</u> |
|          | Committee Member         | Date           |
| Approved | <u>[Signature]</u>       | <u>1-18-73</u> |
|          | Committee Member         | Date           |

Mechanical Engineering Department  
College of Engineering  
Clemson University  
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## ABSTRACT

Using the first and the second derivative of flutter velocity with respect to the parameters, the velocity hypersurface is made quadratic. This greatly simplifies the numerical procedure developed for determining the values of the design parameters such that a specified flutter velocity constraint is satisfied and the total structural mass is near a relative minimum. A search procedure presented utilizes two gradient search methods and a gradient projection method. The procedure is applied to the design of a box beam, using finite-element representation.

The results of the search procedure applied to a box beam indicate that the procedure developed yields substantial design improvement satisfying the specified constraint and does converge to near a local optimum.

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## CHAPTER I

### INTRODUCTION

In recent years numerical programming techniques have grown very relevant in industrial, scientific and military design. This growth has been greatly aided by advances in computer techniques and matrix methods of structural analysis. It has become feasible to apply optimization techniques to design common but complex structures, although most of the development up to this time has been at a research level.

One of the main objectives of optimum design aims at replacing intuitive drafting of a structure by analytical methods so that it may comply with strength conditions. Thus, designing without the theory of optimum design consists of formulation of assumptions and their verification by calculations, whereas the optimum design makes it possible to determine the exact form directly on the basis of given strength conditions.

The major problem with the above mentioned analysis is that, it is time consuming. Most approaches to optimum design are based on iterative methods, and the complex structure has to be analyzed at each iteration to check the various design requirements before proceeding to the next stage of optimum design cycle. This leads us to the following two criteria for optimum design. 1) It is essential that the method of analysis be a rapid one. 2) The number of redesign cycles required to arrive at an optimum should be as small as possible.



### OBJECTIVE

Keeping the above two criteria in mind, in this report the author's aim is to modify an existing optimization scheme to minimize the mass of a structure subject to flutter velocity constraints, as given in (1).

For each iterative cycle an assumed quadratic equation for velocity hypersurface is used. A relatively simple three bay box beam is selected to illustrate the application. The box beam has enough design parameters and degrees of freedom to make the problem meaningful, and if successful can be logically extended to more complicated structures.

In stating the mathematical programming problem it will be assumed that the geometry of the aircraft wing has been fixed in advance. Let  $m$  be the mass of the wing, and  $D_1, D_2, \dots, D_n$  be the design parameters, representing cross sectional areas, plate thicknesses, diameters squared, etc., of the wing. It can be seen that the mass  $m$  is a function of these design parameters, which are selected so that the mass  $m$  is a linear function of these parameters. Now  $n$  side constraints can be placed on these parameters so that they are bounded. Letting  $V$  denote the required minimum flutter velocity, the problem can be expressed as follows:

$$\text{Minimize } m = m(D_1, D_2, \dots, D_p) \quad (1-1)$$

Subject to

$$D_i^l \leq D_i \leq D_i^u, \quad i = 1, p \quad (1-2)$$

and

$$v \geq V \quad (1-3)$$

where  $v$  is the flutter velocity corresponding to the design parameters  $D_1, D_2, \dots, D_p$ . The superscripts  $l$  and  $u$  correspond to the lower and upper bounds, respectively.  $v$  though a function of design parameters, is in general nonlinear.

## CHAPTER II

### FLUTTER

Flutter is defined as an aeroelastic self excited vibration in which the external source of energy is the air stream. The airstream feeds energy into the system by virtue of its position or configuration at least as rapidly as it is dissipated by damping (2, p. 192).

To clarify the above statement let us place an airplane wing in a vacuum. The wing, if disturbed from its equilibrium position, would vibrate in its normal modes. These vibration, because of structural damping, would slowly die out. If however, the wing is moving through air with some constant forward velocity  $v$ , and is suddenly disturbed, as when a gust of wind strikes the wing, then the subsequent motion may be such that the amplitude of vibration will tend to (a) decrease due to damping if the air velocity relative to the wing is less than the critical speed, (b) remain constant, at the critical air speed (also known as the flutter speed of the wing), or (c) increase for a speed higher than the critical air speed, which may at times cause destruction of the structure. Because it is easier mathematically to describe the aerodynamic loads due to simple harmonic motion, theoretical flutter analysis consists of assuming in advance that all dependent displacement variables at flutter speed are proportional to  $e^{i\omega t}$ , where  $\omega$  is the frequency (real) and  $i = \sqrt{-1}$ , and then finding combinations of velocity  $v$  and frequency  $\omega$  for which this actually occurs.

A simple example of flutter as given in Bisplinghoff et. al. (3, p. 528) follows. Consider the case of a rigid, symmetrical airfoil

hinged at its leading edge such that it is elastically restrained from rotating about that edge due to the torsional spring with a spring constant equal to  $K_\alpha$  ft-lb/rad. This is shown in Figure 1.

Let  $I$  = the moment of inertia of the airfoil about the leading edge

$M_y$  = the aerodynamic moment per unit span due to  $\alpha(t)$

$K_\alpha$  = the torsional stiffness of the restraint

The equation of motion for this single-degree-of-freedom system is

$$I_\alpha \frac{d^2 \alpha(t)}{dt^2} + K_\alpha \alpha(t) = M_y. \quad (2-1)$$

The flutter condition is solved for by assuming as a solution

$$\alpha(t) = \alpha_0 e^{i\omega t} \quad (2-2)$$

where  $\alpha_0$  = constant angular amplitude displacement.

The natural torsional frequency  $\omega_\alpha$  is given by

$$\omega_\alpha^2 = \frac{K_\alpha}{I_\alpha} \quad (2-3)$$

Using equations (2-2) and (2-3), and dividing equation (2-1) by  $\pi \rho b^4$ , produces

$$\frac{I_\alpha}{\pi \rho b^4} \left[ -1 + \frac{\omega_\alpha^2}{\omega^2} \right] - \frac{M_y}{\pi \rho b^4 \omega^2 \alpha} = 0$$

where  $b$  = semi-chord,

$\rho$  = density of the air,

hence

$$\frac{I_\alpha}{\pi \rho b^4} \left[ \left( \frac{\omega_\alpha}{\omega} \right)^2 - 1 \right] - m_y = 0 \quad (2-4)$$

$K$  = torsional spring constant  
 $b$  = semichord  
 $\alpha$  = angle of attack  
 $v$  = velocity of airstream

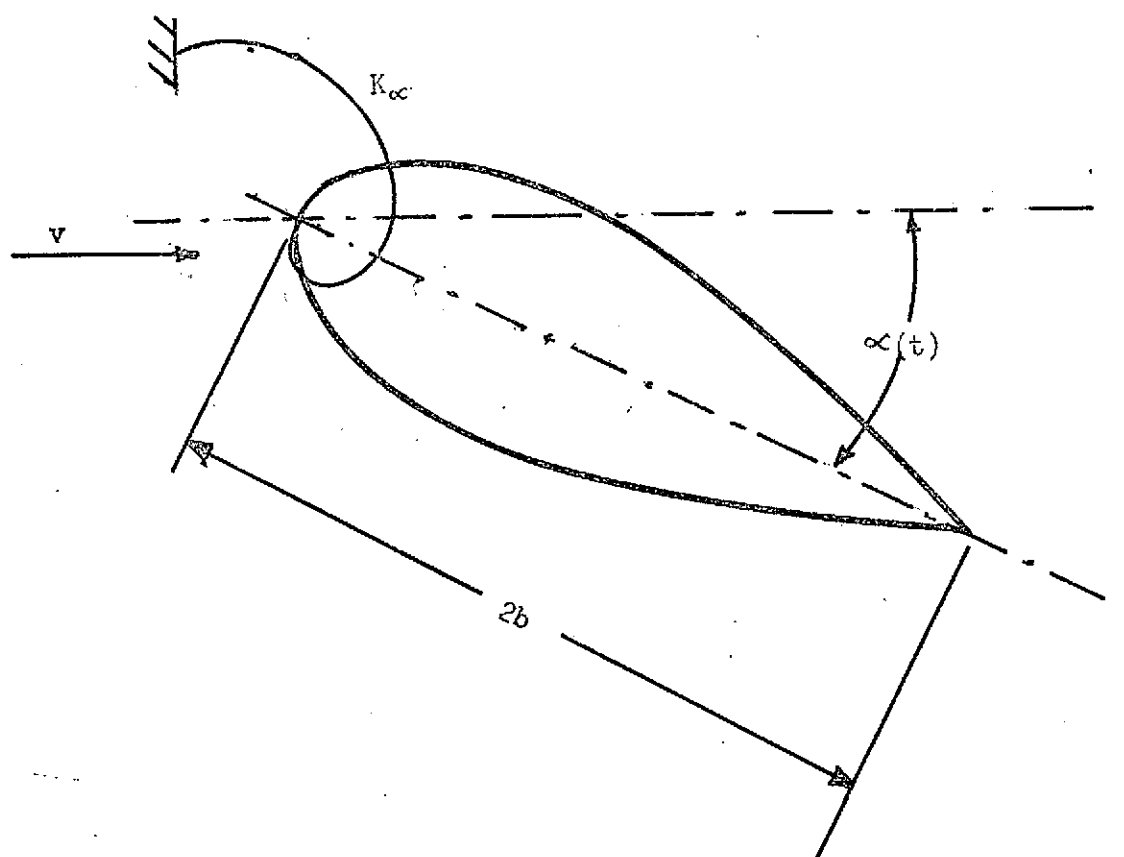


Figure 1. Rigid, Symmetrical Airfoil Flutter  
Restrained about its Leading Edge.

where  $m_y = \frac{M_y}{\frac{1}{4} \rho b^2 \omega^2 \alpha}$  represents the dimensionless aerodynamic coefficient. For a thin airfoil performing small simple harmonic motion in two-dimensional incompressible flow,  $m_y$ , is given by (1, p. 529)

$$m_y = M_\alpha + \frac{1}{2} (L_\alpha + \frac{1}{2}) + \frac{1}{4} L_h$$

where  $M_\alpha$ ,  $L_\alpha$ , and  $L_h$  are complex coefficients which are functions of reduced frequency  $k = \frac{\omega b}{v}$ . Equation (2-4) can be separated into real and imaginary parts. Thus

$$\text{Real } \{m_y\} = \frac{I_\alpha}{\pi \rho b^4} \left[ \left( \frac{\omega_\alpha}{\omega} \right)^2 - 1 \right], \quad (2-5a)$$

and

$$\text{Imaginary } \{m_y\} = 0. \quad (2-5b)$$

Thus, the flutter occurs at that value of reduced frequency  $k$  which just makes the out-of-phase component of the aerodynamic moment vanish, provided the corresponding in-phase part is of such magnitude that equation (2-5a) yields a real flutter frequency  $\omega$  (3, p. 529).

To see how the dynamic instability is caused by the energy added by the airstream, consider the work done by the airstream on the simple airfoil of Figure 1 as it undergoes simple harmonic motion. Since the physical quantities are represented by the real parts of the complex representation,

$$dW = \text{Real } \{M_y\} \times \text{Real } \frac{d\alpha}{dt} dt$$

where  $d\alpha$  = differential increment in  $\alpha(t)$

and  $dW$  = incremental work due to  $d\alpha$ .

Therefore, the total work done during one cycle is

$$W = \int_0^{\frac{2\pi}{\omega}} \text{Real } \{M_y\} \times \text{Real } \frac{d\alpha}{dt} dt. \quad (2-6)$$

Now  $M_y = \pi \rho b^4 \omega^2 \alpha m_y$ ;  $m_y$  is complex

$$= \pi \rho b^4 \omega^2 (m_{yR} + i m_{yI}) \alpha_o e^{i\omega t}$$

where the subscripts R and I denote real and imaginary parts, respectively.

Let 
$$\psi = \tan^{-1} \left( \frac{m_{yI}}{m_{yR}} \right),$$

then

$$M_y = \pi \rho b^4 \omega^2 \left( \sqrt{m_{yR}^2 + m_{yI}^2} \right) e^{i\psi} \alpha_o e^{i\omega t}.$$

Hence

$$\begin{aligned} \text{Real } \{M_y\} &= \pi \rho b^4 \omega^2 \left( \sqrt{m_{yR}^2 + m_{yI}^2} \right) \alpha_o \cos(\omega t + \psi) \\ &= C \cos(\omega t - \psi) \end{aligned} \quad (2-7)$$

where  $C = \pi \rho b^4 \omega^2 \left( \sqrt{m_{yR}^2 + m_{yI}^2} \right) \alpha_o$ .

Now,  $\frac{d\alpha}{dt} = i\omega \alpha_o e^{i\omega t}$

$$= \alpha_o \omega (-\sin \omega t + i \cos \omega t),$$

therefore,

$$\text{Real } \frac{d\alpha}{dt} = -\alpha_o \omega \sin \omega t. \quad (2-8)$$

Substituting equations (2-7) and (2-8) in equation (2-6)

$$\begin{aligned}
 W &= -C_{\alpha_0} \omega \int_0^{\frac{2\pi}{\omega}} \cos(\omega t + \psi) \sin \omega t \, dt \\
 &= -\frac{C_{\alpha_0} \omega}{2} \int_0^{\frac{2\pi}{\omega}} [\sin(2\omega t + \psi) - \sin \psi] \, dt,
 \end{aligned}$$

then

$$W = C_{\alpha_0} \pi \sin \psi$$

or

$$W = \pi \alpha_0 \operatorname{Imaginary} \left\{ \frac{M_y}{e^{i\omega t}} \right\}. \quad (2-9)$$

Since  $\frac{M_y}{e^{i\omega t}}$  is independent of time, the sign of equation (2-9) depends upon the aerodynamic coefficients  $M_{\alpha}$ ,  $L_{\alpha}$ , and  $L_h$ . A negative sign would mean that the airstream is extracting work from the elastic system, thus providing aerodynamic damping. A positive sign would mean that the airstream is adding energy to the system and would cause the airfoil in this simple example to flutter.



## CHAPTER III

### WORK BY OTHERS

Although the technique for analytical prediction of aeroelastic phenomena has been available for a long time, to date there have been relatively few published works dealing with optimization under aeroelastic constraints, since most of the work in optimization has dealt with the conventional conditions of strength, stiffness and stability.

Rudisill and Bhatia (1) have developed a numerical procedure for determining the values of the design parameters such that a specified flutter velocity constraint is satisfied and the total structural mass is near a relative minimum. The search procedure utilized two gradient search methods and a gradient projection method. In the above procedure substantial design improvement was made but convergence to an optimum was not obtained in a reasonable computer execution time. Since then Rudisill and Bhatia (4) have obtained an analytical solution for the second partial derivatives of the eigenvalues of the flutter equation along with the equation for finding the second partial derivatives of a flutter velocity of an aircraft structure with respect to the structural parameters. Using these partial derivatives in computing the step size used in the projected gradient search along a constant mass hyperplane has helped in cutting down on the number of redesign cycles to arrive at the optimum.

Cooper (5) in his report, has attempted a direct method of solution for the flutter velocity. He also has applied his method to the optimization of a cantilevered box beam for minimum mass due

to a flutter velocity constraint. His method of solution required a plot of the imaginary part of the eigenvalue of the flutter equation versus the dependent variable of the flutter equation, i.e. velocity divided by the circular frequency. From this plot the crossover points on the dependent variable axis are sought. From these points the lowest or the critical velocity for which the structure will have divergent oscillations may be calculated. He has simplified his procedure by fitting a simple quadratic or cubic equation for the required plots, and has obtained good results.

Siegel (6) within the last year has developed an optimization method for accurately and rapidly calculating, through a completely automated digital computer program, the minimum weight spanwise distribution of an airfoil surface to provide a given required flutter speed. This program starts by calculating the flutter speed for a configuration having adequate strength. If the calculated flutter speed is lower than required an automatically determined increment of structural material is added to the spanwise location where the strain energy per unit structural volume in the flutter mode is a maximum. Using the new structural data the flutter speed is again calculated. This process is repeated automatically within the computer until the required flutter speed is attained. This method is thus absolutely based on the concept that the most efficient structure is one that has constant strain energy per unit structural volume in the flutter mode.

In this report the method developed by Rudisill and Bhatia (1,4) has been extended to include a curve fitting technique for the velocity hyperplanes, and other minor changes to accommodate it have been made, thus making the process efficient.

## CHAPTER IV

### DESCRIPTION OF OPTIMIZATION PROCEDURE

If stiffness distributions are not properly optimized for flutter, aircraft structures can become significantly heavier than necessary. For the objective formulated in Chapter 1, there are numerous search strategies that could be devised to attempt a solution to an optimization problem.

The optimization procedures developed here are basically a gradient search method, the greatest rate of improvement of the function being found by moving along the gradient. (The derivatives of an objective function with respect to each of the  $n$  parameters are collectively called the gradients of the objective function.)

The present optimization procedure (4) utilizes three well known gradient search routines. A simplified graphical illustration of their behavior is shown in Figure 3. In the following three sections the searches are individually described; later on in this chapter the curve fitting method for the velocity hyperplane is given.

Before we go into the mathematical derivation, a few important assumptions are stated.

(i) the search is to be conducted in the  $\bar{P}$  dimensional space of design parameters  $D_1, D_2, \dots, D_{\bar{P}}$ . The column matrix  $\{D(p)\}$  defines a design point  $p$  in this space which corresponds to a particular structure configuration such that  $D_i > 0, i = 1, \bar{P}$ ,

(ii) the flutter velocity is uniquely defined at any point  $\{D\}$ .

Thus,  $v$  which is assumed to be a continuous function of  $\{D\}$  is a

scalar point function. The total mass  $m$  is also a continuous scalar function of  $\{D\}$ .

Let  $p$  and  $p^*$  be two neighboring points defined by  $\{D(p)\}$  and  $\{D(p^*)\}$ , respectively. The scalar distance  $\Delta s$  between these points is

$$\Delta s = |\{D(p)\} - \{D(p^*)\}| = \sqrt{\sum_{i=1}^P (\Delta D_i)^2} \quad (4-1)$$

It is now assumed that the limits of the difference quotients of  $v$  and  $m$  as  $\Delta s$  tends to zero, exist. Thus

$$\begin{aligned} \text{Limit}_{\Delta s \rightarrow 0} \frac{v(p^*) - v(p)}{\Delta s} &= \frac{dv}{ds}, \\ \text{Limit}_{\Delta s \rightarrow 0} \frac{m(p^*) - m(p)}{\Delta s} &= \frac{dm}{ds} \end{aligned} \quad (4-2)$$

Equation (4-2) defines the directional derivatives of  $v$  and  $m$ .

#### VELOCITY GRADIENT SEARCH

This routine is employed in order to increase the velocity. In this routine the search moves perpendicular to the velocity contours, i.e. maximum increase in velocity. The desired velocity is reached in one or more steps in an iterative fashion.

The maximum rate of change of velocity is given by the normal derivative and is equal to the absolute value of the gradient vector of the velocity. It may be expressed as

$$\frac{dv}{dn} = \sqrt{\sum_{j=1}^P \left( \frac{\partial v}{\partial D_j} \right)^2} \quad (4-3)$$

A new velocity along a gradient curve corresponding to small design parameter changes may be approximated by the expression

$$V \approx v + \frac{dv}{dn} \Delta n \quad (4-4)$$

where  $V$  and  $v$  are the new and old velocities respectively, and  $\Delta n = \Delta s$  along the normal vector to constant velocity hypersurface at  $\{D\}$ . The direction cosines of the gradient are

$$l_i = \frac{\frac{\partial v}{\partial D_i}}{\frac{dv}{dn}} \approx \frac{\Delta D_i}{\Delta n}, \quad i = 1, P. \quad (4-5)$$

Therefore,

$$\Delta D_i \approx \frac{\frac{\partial v}{\partial D_i} \Delta n}{\frac{dv}{dn}}, \quad i = 1, P. \quad (4-6)$$

From equation (4-4)

$$\Delta n \approx \frac{(V - v)}{\frac{dv}{dn}}. \quad (4-7)$$

Substituting equation (4-3) and (4-7) in equation (4-6)

$$\Delta D_i \approx \frac{(V - v) \frac{\partial v}{\partial D_i}}{\sum_{j=1}^P \left( \frac{\partial v}{\partial D_j} \right)^2}, \quad i = 1, P. \quad (4-8)$$

Thus the new value of design parameters  $D_i^*$ ,  $i = 1, P$ , are given by

$$D_i^* \approx D_i + \frac{(V - v) \frac{\partial v}{\partial D_i}}{\sum_{j=1}^P \left( \frac{\partial v}{\partial D_j} \right)^2}, \quad i = 1, P. \quad (4-9)$$

Equation (4-9) will yield approximate values of the design parameters corresponding to  $V$ .

Since the velocity is a nonlinear function of the design parameters, the true velocity corresponding to the set of design parameters  $D^*$  will not be equal to  $V$ . An iterative procedure may be used to determine the set of design parameters corresponding to the desired velocity. The search was programmed such that  $D_i$ ,  $i = 1, P$ , in equation (4-8) is either positive or zero.

#### MASS GRADIENT SEARCH

This routine is used in order to reduce the velocity. In this routine the search moves perpendicular to the constant mass hyperplanes, i.e. the direction of the maximum rate of decrease in mass. This process is repeated until the velocity is less than or equal to the desired velocity.

The total differential of the velocity may be expressed as

$$dv = \sum_{j=1}^P \frac{\partial v}{\partial D_j} dD_j. \quad (4-10)$$

The normal derivative of the total mass  $m$  may be expressed by the equation

$$\frac{dm}{dn} = \sqrt{\sum_{j=1}^P \left( \frac{\partial m}{\partial D_j} \right)^2} \quad (4-11)$$

and the direction cosines of the mass gradient vector are given by the relation

$$\lambda_i = \frac{\frac{\partial m}{\partial D_i}}{\frac{dm}{dn}} \approx \frac{\Delta D_i}{\Delta n}, \quad i = 1, P \quad (4-12)$$

where  $\Delta n = \Delta s$  along the normal vector to constant mass hypersurface at  $\{D\}$ . Approximating the total differential  $dv$  by increment  $v$  and substituting  $\Delta D_i$  from equation (4-12) in equation (4-10) yields

$$\Delta v \approx \frac{\sum_{j=1}^P \left( \frac{\partial v}{\partial D_j} \frac{\partial m}{\partial D_j} \right) \Delta n}{\frac{dm}{dn}}$$

or

$$\Delta n \approx \frac{\sqrt{\sum_{i=1}^P \left( \frac{\partial m}{\partial D_i} \right)^2} \Delta v}{\sum_{j=1}^P \frac{\partial v}{\partial D_j} \frac{\partial m}{\partial D_j}} \quad (4-13)$$

Substituting equation (4-13) into equation (4-12) yields

$$\Delta D_i \approx \frac{\frac{\partial m}{\partial D_i} \Delta v}{\sum_{j=1}^P \frac{\partial v}{\partial D_j} \frac{\partial m}{\partial D_j}} \quad (4-14)$$

The new set of design parameters  $\{D^*\}$  can again be computed from

$$D_i^* = D_i + \Delta D_i$$

It should be noted that  $\Delta v = V - v$ .

The mass gradient search is also an iterative procedure like the velocity gradient search. The search was programmed in such a way that  $\Delta D_i$ ,  $i = 1, P$ , in equation (4-14) would be either negative or zero.



### GRADIENT PROJECTION SEARCH

This routine is employed in order to reach a relative maximum of the velocity while the mass is held constant. The parameters are varied such that the search proceeds tangent to a constant mass hypersurface in the direction of the maximum rate of increase of velocity.

The gradient projection method allows variations of design parameters to be taken so as to satisfy the behavior constraints at all times. The search for an optimum cannot therefore proceed in the steepest ascent direction, but must always be restricted to movements satisfying the constraints. Here the velocity  $v$  will be maximized while the total mass  $m$  is held constant.

With the aid of the second derivatives of the flutter velocity with respect to the design parameters, and the projected gradient search as given in [7], the step size  $s$  could be expressed as

$$s = - \frac{\sum_{j=1}^P \frac{\partial v}{\partial D_j} \frac{dD_j}{ds}}{\sum_{j=1}^P \sum_{k=1}^P \frac{\partial^2 v}{\partial D_j \partial D_k} \frac{dD_j}{ds} \frac{dD_k}{ds}}, \quad (4-15)$$

where  $\frac{dD_j}{ds}$  are the direction cosines corresponding to the direction of the maximum rate of increase of the flutter velocity along a constant mass hypersurface. These direction cosines [7] are,

$$\frac{dD_j}{ds} = \frac{1}{2\lambda_0} \left( \frac{\partial v}{\partial D_j} + \lambda_1 \frac{\partial m}{\partial D_j} \right) \quad (4-16)$$

where  $2\lambda_0$  and  $\lambda_1$  are given as

$$\lambda_1 = - \frac{\sum_{j=1}^n \frac{\partial m}{\partial D_j} \frac{\partial v}{\partial D_j}}{\sum_{j=1}^n \left(\frac{\partial m}{\partial D_j}\right)^2} \quad (4-17)$$

$$2\lambda_0 = \left\{ \sum_{j=1}^n \left[ \left(\frac{\partial v}{\partial D_j}\right)^2 + \frac{\partial v}{\partial D_j} \lambda_1 \frac{\partial m}{\partial D_j} \right] \right\}^{1/2} \quad (4-18)$$

and  $m$  is the total mass of the structure. New parameters may be computed from the relation

$$D_j^* = D_j + \left( \frac{\partial v}{\partial D_j} + \lambda_1 \frac{\partial m}{\partial D_j} \right) S/2\lambda_0 \quad (4-19)$$

If the total mass is formulated as a linear function of the design parameters then the search will always be along a constant mass hyperplane except those times when the side constraints for the design parameters are encountered. In that case the search might completely fail, i.e. velocity decreases instead of increasing. When this is encountered a different approach is used which is described later on in the computation procedure.

#### CURVE FITTING FOR VELOCITY

In the optimization procedure developed here, we are trying to minimize the mass of a structure (aircraft) to satisfy flutter velocity requirements. In its simplest form we will formulate the program for a cantilevered box beam, which consists of twelve variable parameters, but could be extended to include any finite number of variables. In

order to make the procedure systematic and more efficient the velocity contours are approximated by a quadratic equation of the form,

$$v = A + \sum_{i=1}^n B_i D_i + \sum_{i=1}^n \sum_{j=1}^n C_{ij} D_i D_j \quad (4-20)$$

The coefficients  $A$ ,  $B_i$ , and  $C_{ij}$  could now be calculated at the known point using the following procedure: Taking the first derivative of equation (4-20) with respect to some parameter  $D_h$

$$\frac{\partial v}{\partial D_h} = B_h + \sum_{j=1}^n C_{hj} D_j + \sum_{i=1}^n C_{ih} D_i \quad (4-21)$$

and the second derivative with respect to a different parameter  $D_g$  is

$$\frac{\partial^2 v}{\partial D_h \partial D_g} = C_{hg} + C_{gh} \quad (4-22)$$

Now assuming  $C_{hg} = C_{gh}$ , the above equation yields

$$C_{hg} = \frac{1}{2} \frac{\partial^2 v}{\partial D_h \partial D_g} \quad (4-23)$$

Substituting this in equation (4-21)

$$B_h = \frac{\partial v}{\partial D_h} - \sum_{j=1}^n C_{hj} D_j - \sum_{i=1}^n C_{ih} D_i \quad (4-23)$$

or,

$$B_h = \frac{\partial v}{\partial D_h} - 2 \sum_{i=1}^n C_{ih} D_i$$

Since  $C$  is symmetric, we have

$$B_h = \frac{\partial v}{\partial D_h} - \sum_{i=1}^n \frac{\partial^2 v}{\partial C_i \partial D_h} D_i \quad (4-24)$$

Again, equation (4-20) could be written

$$A = v - \sum_{j=1}^n B_j D_j - \sum_{i=1}^n \sum_{j=1}^n C_{ij} D_i D_j$$

Substituting values of B's and C's from (4-24) and (4-23) we have

$$A = v - \sum_{j=1}^n \left[ \frac{\partial v}{\partial D_j} - \sum_{i=1}^n \frac{\partial^2 v}{\partial D_i \partial D_j} D_i \right] D_j - \sum_{j=1}^n \sum_{i=1}^n \frac{1}{2} \frac{\partial^2 v}{\partial D_i \partial D_j} D_i D_j \quad (4-25)$$

The first and second derivatives of flutter velocity with respect to the design parameters is given in (1,4), the coefficient's  $C_{hg}$ ,  $B_h$ , and A are then calculated with the aid of equations (4-23 through 4-25). These coefficients are recomputed at the beginning of each new design cycle.

#### COMPUTER PROCEDURE

A simplified flow diagram of the optimization procedure is shown in Figure 2.  $\epsilon_v$  and  $\epsilon_m$  are the specified tolerances used to compare the computed velocity  $v$  and the computed mass TMASS to the desired velocity  $V$  and the previous mass TMASS1, respectively, at various stages of the optimization procedure.

Initial parameters are assumed, an initial velocity is computed and is fitted to an assumed quadratic surface whose coefficients A,  $B_h$ , and  $C_{hg}$  are evaluated by the method described in the last section.

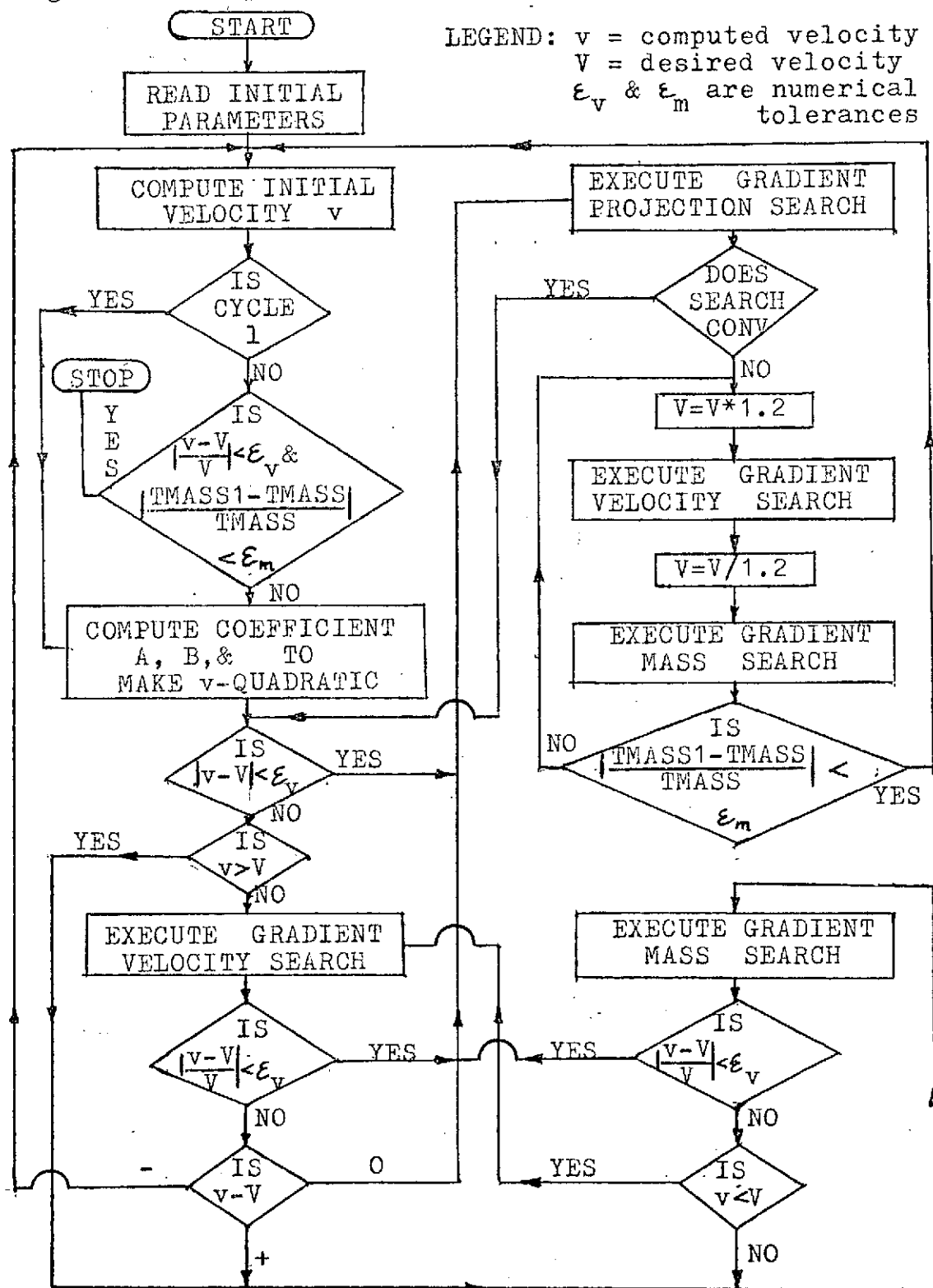
As shown in Figure 3, if the initial velocity is not within tolerance  $\epsilon_v$ , and if  $v$  is greater than  $V$ , then the gradient mass search is executed as shown from a to b. If instead,  $v$  is less than  $V$ , as at point a', then the gradient velocity search a' to b' is

executed. The gradient mass search reduces the mass, but may either increase or decrease the velocity; the gradient velocity search increases the velocity.

When the velocity is within tolerance  $\epsilon_v$ , then a gradient projection search is executed along a constant mass hyperplane for example from c to d. If any side constraints are encountered during the search, the appropriate design parameters are set equal to their constraint value, and the search deviates from the constant mass hyperplane. If the computed velocity increases from its previous value, the search is continued as shown in Figure 3 in an iterative fashion until the flutter velocity reaches a maximum for that constant mass hyperplane.

If a large number of side constraints become active, then the projected gradient search will deviate greatly from the constant mass hyperplane. At that stage the search fails and an alternate procedure as shown in Figure 4 is employed. In this procedure, the gradient velocity search and the gradient mass search are alternated in the following manner. For the desired velocity  $V$  the gradient mass search proceeds from a to b Fig. 4, now the desired velocity is changed to  $1.2V$  and then utilizing the gradient velocity search, a step is made from b to c. This procedure is repeated a number of times until the change in mass (point 0) is within tolerance. The search is terminated if the change in the mass for any cycle is less than some small prescribed number.

Figure 2 - Simplified Flow Diagram of Optimization Procedure



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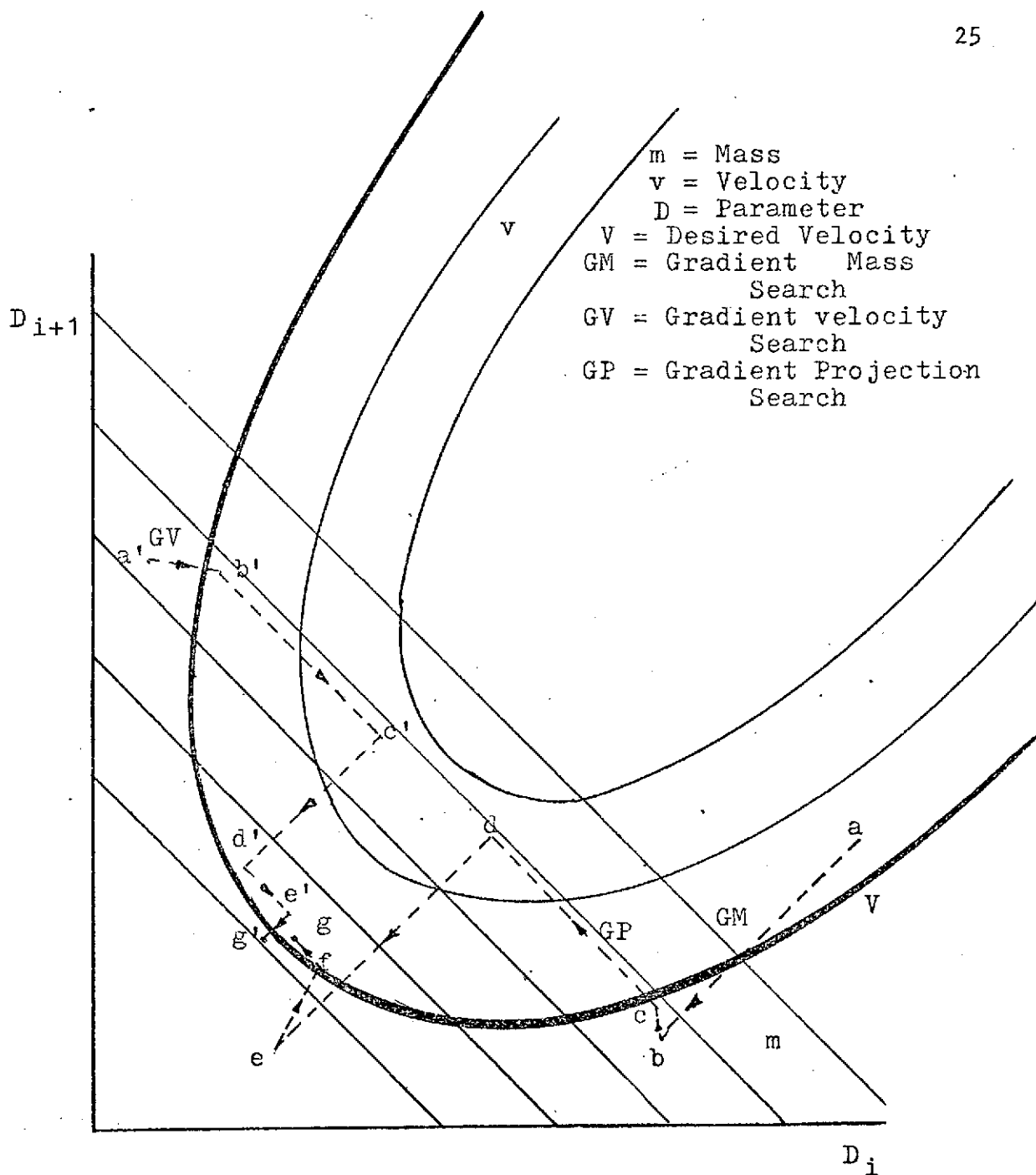


Figure 3 - Simplified Assumed Curves Representing  
the Direction in which Different Search  
Proceeds

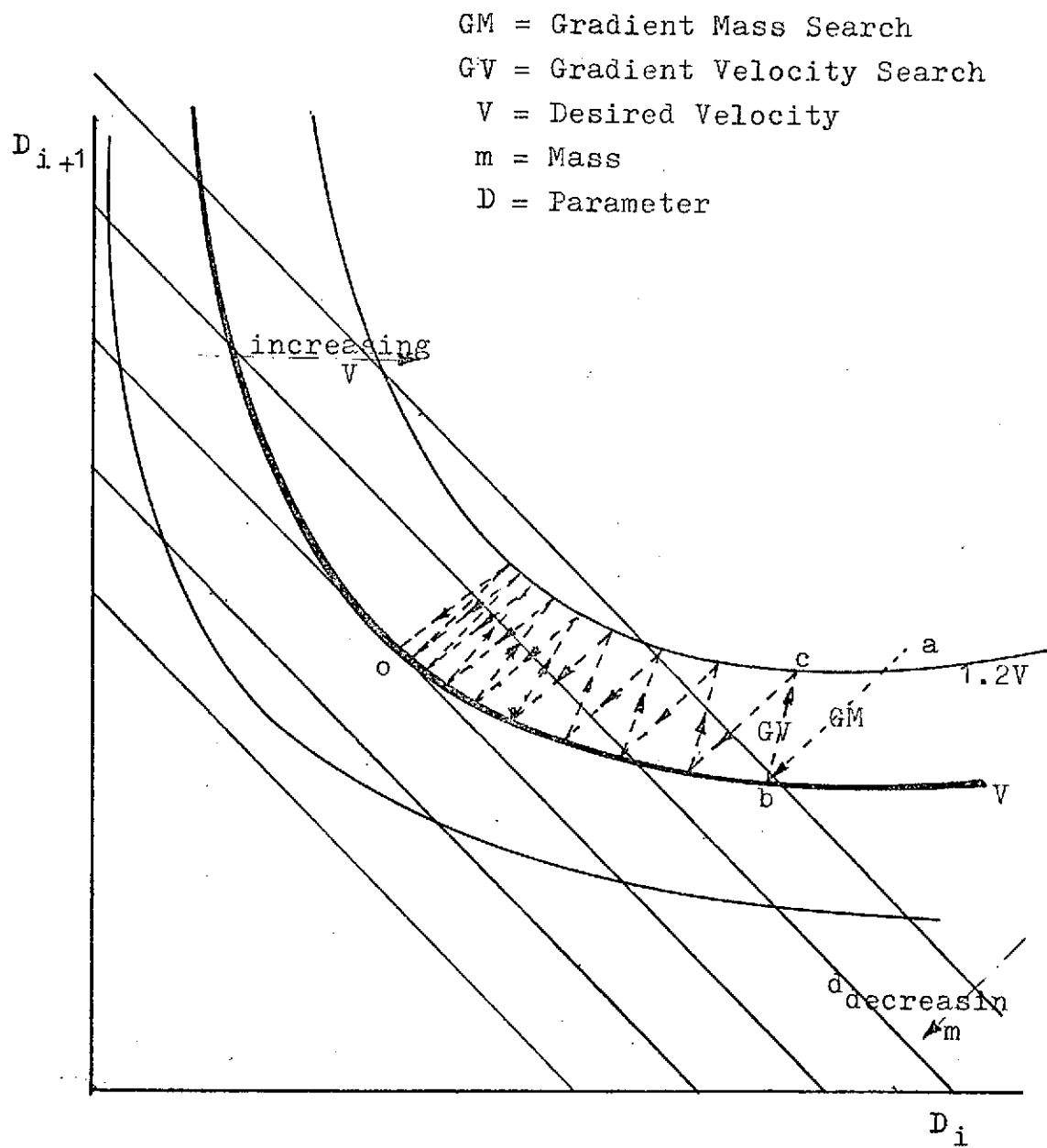


Figure 4 - Alternate Search when Gradient Projection Fails



## CHAPTER V

### OPTIMIZATION OF A BOX BEAM

Figure 5 shows a three-bay box beam representing a uniform cantilever aircraft wing structure. It is assumed, as stated in Chapter I, that the wing geometry and shape have been fixed in advance. Thus the height  $H$ , width  $W$  and length of each bay  $L$  are treated as constants, these values could be different for each bay if necessary. The design parameters for each bay are defined to be: (i) area of longitudinals, (ii) front and back web, (iii) top and bottom web thickness and (iv) rib thickness. These parameters were required to be of uniform value for each bay. Since there are three bays, the total number of variable design parameters is twelve. Other constant parameters needed in the analysis are defined in Figure 5. With the design parameters defined as above, the total mass to be minimized is a linear function of the design parameters. Solution to the mathematical programming problem formulated in Chapter I is found for the box beam described above.

A simplified flow diagram explaining the basic logic of the optimization program developed, was explained in Chapter IV (Figure 2). Each of the three gradient search routines calls a sub-routine which, utilizing the established coefficients of the assumed quadratic, returns the computed velocity and the partial derivatives of the velocity with respect to the design parameters. In the flow diagram of Figure 2,  $v$  and  $V$  are the computed and the desired velocities, respectively.

The aerodynamic matrix is formulated from the equation used by Smilg and Wasserman (8, p. 398).

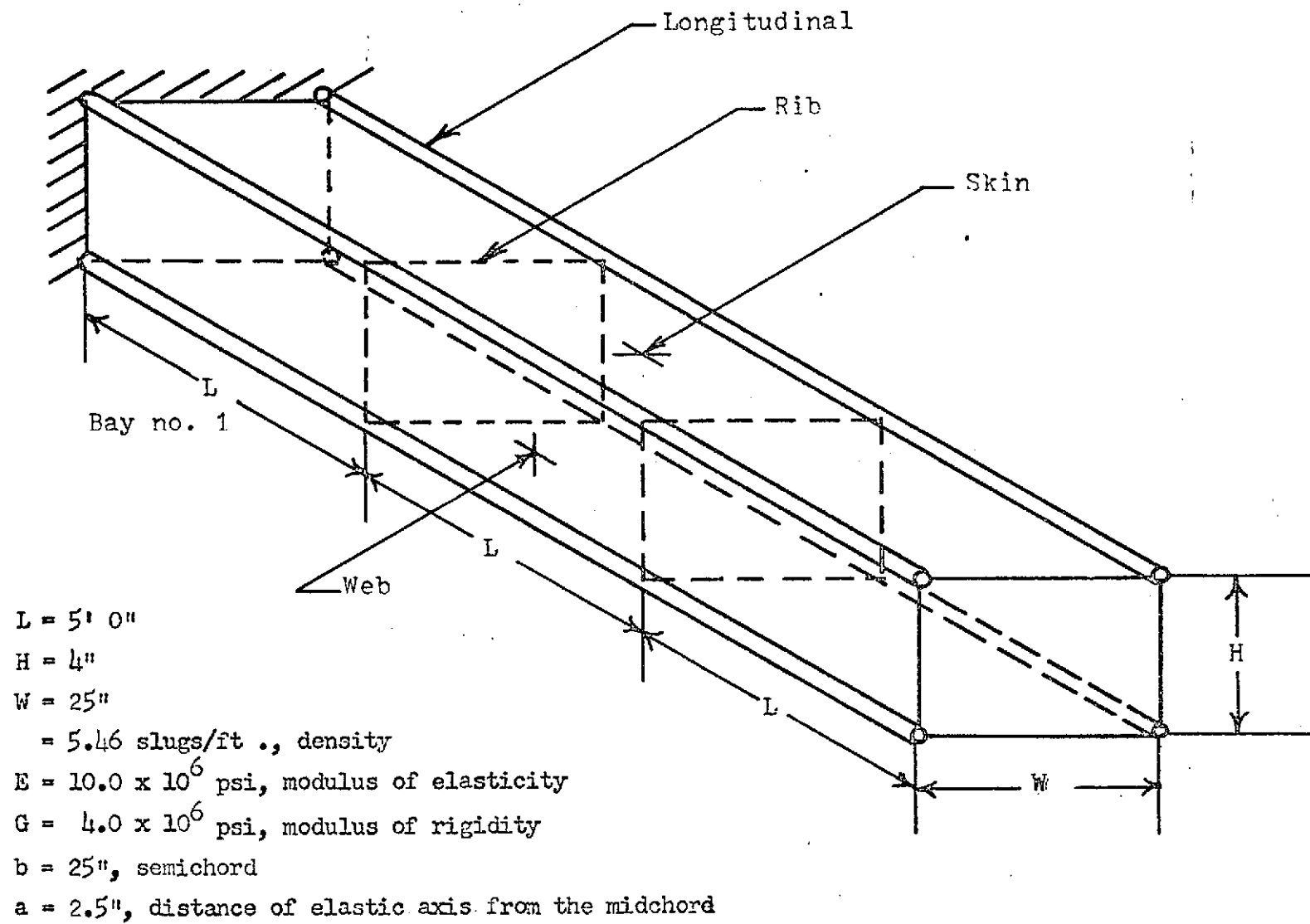


FIGURE 5. Rectangular Three-Bay Box Beam

TABLE I INITIAL DESIGN PARAMETERS & CONSTRAINTS

## LEGEND:

J = Number of bay; 1, 2, 3

D(J) = Area of longitudinals for the  $j^{\text{th}}$  bayD(3+J) = Front and back web thickness for the  $j^{\text{th}}$  bayD(6+J) = Top and bottom skin thickness for the  $j^{\text{th}}$  bayD(9+J) = Rib thickness for the  $j^{\text{th}}$  bay.

| VARIABLE<br>DESIGN<br>PARAMETERS | INITIAL<br>VALUES | CONSTRAINTS |          |
|----------------------------------|-------------------|-------------|----------|
|                                  |                   | UPPER       | LOWER    |
| D(1), Sq. in.                    | 0.01389           | 0.1389      | 0.002315 |
| D(2), Sq. in.                    | 0.01389           | 0.1389      | 0.002315 |
| D(3), Sq. in.                    | 0.01389           | 0.1389      | 0.002315 |
| D(4), in.                        | 0.006667          | 0.06667     | 0.001111 |
| D(5), in.                        | 0.006667          | 0.06667     | 0.001111 |
| D(6), in.                        | 0.006667          | 0.06667     | 0.001111 |
| D(7), in.                        | 0.003333          | 0.03333     | 0.000555 |
| D(8), in.                        | 0.003333          | 0.03333     | 0.000555 |
| D(9), in.                        | 0.003333          | 0.03333     | 0.000555 |
| D(10), in.                       | 0.003333          | 0.03333     | 0.000555 |
| D(11), in.                       | 0.003333          | 0.03333     | 0.000555 |
| D(12), in.                       | 0.003333          | 0.03333     | 0.000555 |

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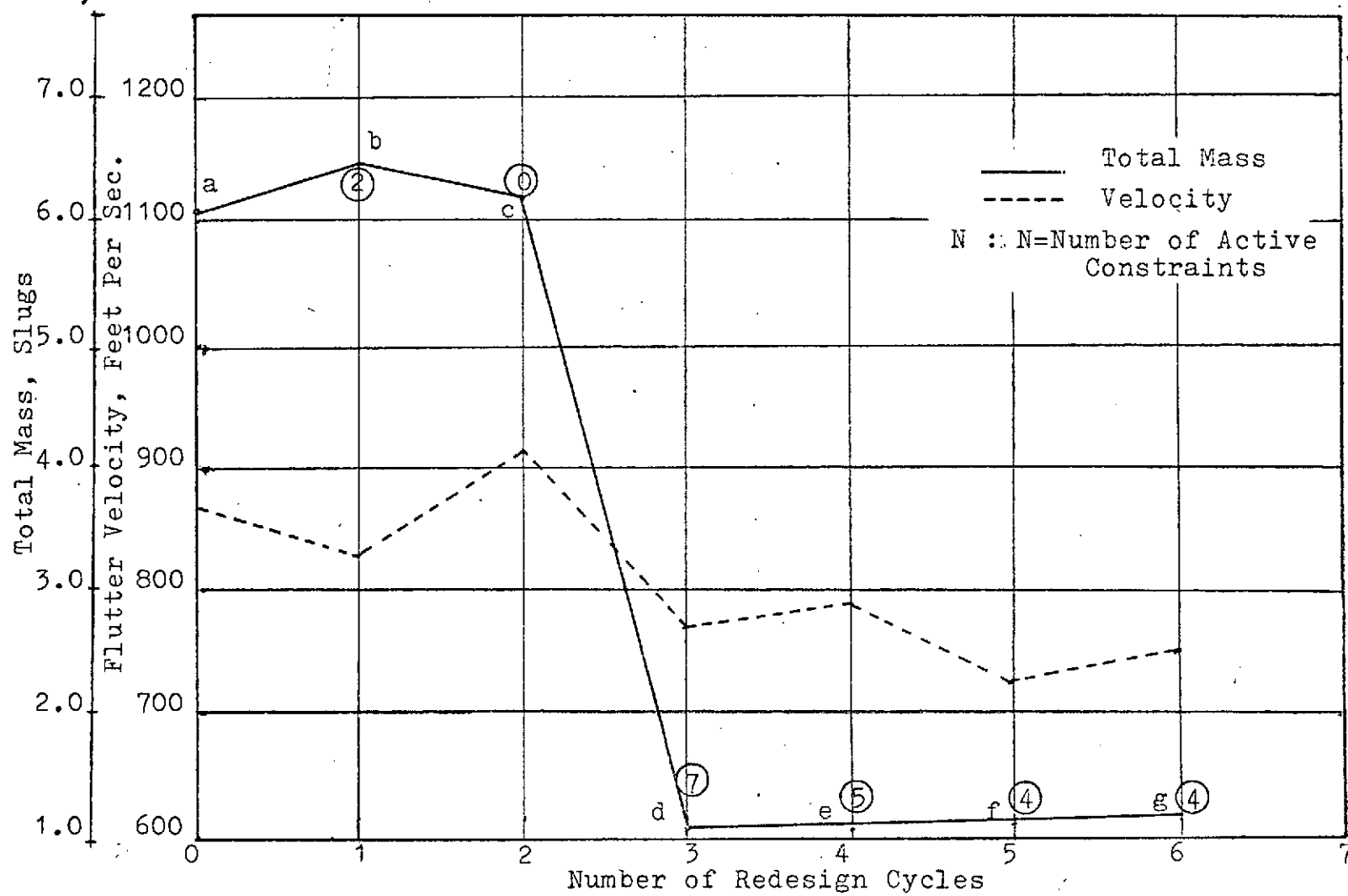


Figure 6 - Flutter Optimization.

TABLE II. RESULTS OF FLUTTER OPTIMIZATION

## LEGEND:

J = Number of bay; 1, 2, 3

D(J) = Area of longitudinals for the  $j^{\text{th}}$  bayD(3+J) = Front and back web thickness for the  $j^{\text{th}}$  bayD(6+J) = Top and bottom skin thickness for the  $j^{\text{th}}$  bayD(9+J) = Rib thickness for the  $j^{\text{th}}$  bay

| VARIABLE<br>DESIGN<br>PARAMETERS | INITIAL<br>VALUES | FINAL<br>VALUES |
|----------------------------------|-------------------|-----------------|
| D(1), Sq. in.                    | 0.01389           | 0.002554        |
| D(2), Sq. in.                    | 0.01389           | 0.002315*       |
| D(3), Sq. in.                    | 0.01389           | 0.002315*       |
| D(4), in.                        | 0.006667          | 0.001471        |
| D(5), in.                        | 0.006667          | 0.001476        |
| D(6), in.                        | 0.006667          | 0.001111*       |
| D(7), in.                        | 0.003333          | 0.000555*       |
| D(8), in.                        | 0.003333          | 0.0008296       |
| D(9), in.                        | 0.003333          | 0.0005988       |
| D(10), in.                       | 0.003333          | 0.001232        |
| D(11), in.                       | 0.003333          | 0.001032        |
| D(12), in.                       | 0.003333          | 0.001130        |
| VELOCITY,<br>FEET PER SECOND     | 870.1             | 747.3           |
| MASS,<br>SLUGS                   | 6.099             | 1.098           |

\*Parameter equal to the lower constraint

The three gradient search procedures developed use partial derivatives of total mass and also the first and second partials of velocity with respect to the design parameters. When the total mass is a linear function of the design parameters, the partial derivatives of the total mass are constant and need to be evaluated only once. If the total mass is not a linear function of the parameters, they could be easily evaluated at each step in the design space. The partial derivatives of the flutter velocity are computed as shown in reference (1,4).

The initial variable parameters used for the box beam, and the upper and lower side constraints are given in Table I.

The total mass of the beam and the flutter velocity versus the number of redesign cycles are plotted in Figure 6. The desired flutter velocity was specified as 800 feet per second. The design was started with an initial velocity of 870.1 feet per second and a mass of 6.099 slug (a, Fig. 6). The velocity was first decreased utilizing the gradient mass search, and then increased by the gradient projection search, holding mass constant. These two steps were again repeated but the gradient projection search failed. An alternate procedure as described in Chapter IV (Figure 4) was used, and the cycle terminated due to no change in mass (b, Fig. 6). The second cycle started with an initial velocity of 388 feet per second. The gradient velocity search was used for step one and then the search iterated between the gradient projection and the gradient mass search, but did not converge in the maximum number of allowed steps. Starting with a velocity of 724 feet

per second and utilizing all three searches it converged to the mass of 1.047 slugs and a velocity of 770.8 feet per second (d, Fig 6), indicating that the program had probably produced nearly a local optimum. Three more cycles were carried out with a very small change in mass, until finally the search terminated at a velocity of 743.7 feet per second and a mass of 1.098 slugs. Thus a local optimum was reached in six cycles, giving the best results so far.

The flutter velocity was computed by solving the complex eigenvalue problem expressed in the fundamental flutter equation (1). A computer program for solving the complex eigenvalue problem was obtained from the National Aeronautics and Space Administration.

Table II shows the results of the optimization study and lists the initial and the final design parameters and the corresponding mass and velocity.

An IBM 370, Model 155 computer at the Clemson University Computer Center was used. The total execution time, as recorded by the central processing unit (CPU) was 112 seconds for 6 redesign cycles carried out by one computer run. Comparing it with a previous search which took 123 seconds, there is not much saving in time, but this procedure may be much more efficient when used with system which has a much larger number of parameters and degrees of freedom.

## CONCLUSIONS

The quadratic curve fitting for the velocity hypersurface is the important feature of this work. The time required for optimizing the cantilevered box beam by the method developed here is less than the previous method.

The optimization scheme used is based on three simple gradient search techniques, this search have been modified so as to estimate step size in case of overshoot. The scheme could be extended to include other behavioral constraints such as stresses, displacements, and buckling. Since it did result in substantial improvement for a system with 12 design variables, the author feels that it is reasonable to expect that this scheme would at least work with a similar degree of success for a system with larger number of design variables, and in fact, might be more efficient.

Also, there is much room for innovation in the use of search schemes which apply other available gradient search methods or their combinations to the problems of this nature.



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